Feshbach resonance and mesoscopic phase separation near a quantum critical point in multiband FeAs-based superconductors

Supercond. Sci. Technol. **22** (2009) 014004 (12pp) doi:10.1088/0953-2048/22/1/014004

Feshbach resonance and mesoscopic phase separation near a quantum critical point in multiband FeAs-based superconductors

Rocchina Caivano[1], Michela Fratini[1], Nicola Poccia[1], Alessandro Ricci[1], Alessandro Puri[1], Zhi-An Ren[2], Xiao-Li Dong[2], Jie Yang[2], Wei Lu[2], Zhong-Xian Zhao[2], Luisa Barba[3] and Antonio Bianconi[1]

[1] Department of Physics, Sapienza University of Rome, Piazzale Aldo Moro 2, 00185 Roma, Italy
[2] Laboratory for Superconductivity, Institute of Physics and Beijing National Laboratory for Condensed Matter Physics, Chinese Academy of Sciences, PO Box 603, Beijing 100190, People's Republic of China
[3] Institute of Crystallography, National Council of Research, Elettra, 34012 Trieste, Italy

Received 1 October 2008
Published 16 December 2008
Online at stacks.iop.org/SUST/22/014004

Abstract

High-T_c superconductivity in FeAs-based (pnictide) multilayers, evading temperature decoherence effects in a quantum condensate, is assigned to a Feshbach resonance (also called shape resonance) in the exchange-like interband pairing. The resonance is switched on by tuning the chemical potential at an electronic topological transition (ETT) near a band edge, where the Fermi surface topology of one of the subbands changes from one-dimensional (1D) to two-dimensional (2D) topology. We show that the tuning is realized by changing (i) the misfit strain between the superconducting planes and the spacers, (ii) the charge density, and (iii) the disorder. The system is at the verge of a catastrophe, i.e. near a structural and magnetic phase transition associated with the order-to-disorder phase transition of the stripes (analogous to the 1/8 stripe phase in cuprates). Fine tuning of both the chemical potential and the disorder pushes the critical temperature T_s of this phase transition to zero, giving a quantum critical point. Here the quantum lattice and magnetic fluctuations promote the Feshbach resonance of the exchange-like anisotropic pairing. This superconducting phase that resists the attacks of temperature is shown to be controlled by the interplay of the hopping energy between stripes and the quantum fluctuations. The superconducting gaps in the multiple Fermi surface spots reported by the recent ARPES experiment of Evtushinsky *et al* (2008 arXiv: 0809.4455) are shown to support the Feshbach scenario.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The discovery of high-temperature superconductivity in FeAs multilayered materials [1–10] provides a new system where the unknown quantum mechanism for evading temperature decoherence effects in a macroscopic quantum condensate is active. In fact, in these FeAs-based materials the superconducting quantum coherent phase of a fermionic gas resists at temperatures higher than the liquid hydrogen boiling temperature like in doped cuprate perovskites and diborides. It is possible that there is a common quantum mechanism active in all these materials; therefore, the research is looking for similarities of the normal and superconducting phase between these systems. A common structural characteristic of cuprates, diborides, and FeAs-based superconductors is the heterostructure at the atomic limit: a superlattice of metallic layers with strong covalent bonds (atomic CuO_2 bcc layers or atomic graphene-like B_2 monolayers, or molecular $FeAs_4$

layers) intercalated by spacers made of different materials with a different electronic structure (fcc rocksalt layers like La_2O_2 [11, 12], or hcp Mg/Al layers, or rare-earth oxide layers, respectively) [13].

There are different types of layered FeAs-based superconductor that have an analogous structure.

(a) Doped quaternary rare-earth iron oxypnictides, ROFePn (R = rare-earth metal and Pn = pnicogen = O) (R = La, Pr, Nd, Ce, Sm, ...) made of FeAs layers intercalated by RO oxide layers [1–10]. These '1111' systems at room temperature have a tetragonal (space group $P4/nmm$) structure. What is critical to the high-T_c superconductivity (55 K is the maximum T_c) is the F substitution for oxygen (15–20 at.%), called electron doping (n-type) of the formal $[FeAs]^{-1}$ layers; or Sr^{2+} for R^{3+} doping (4–13 at.%), called hole doping (p-type); or the introduction of oxygen defects.

(b) Doped alkaline-earth iron arsenides, $AeFe_2As_2$ (Ae = Sr, Ba), made of $[Fe_2As_2]^{-2}$ layers separated by simple Ae layers, which have a tetragonal $ThCr_2Si_2$-type structure, space group $I4/mmm$, called '122'. They become superconductors (38 K maximum T_c) with appropriate substitution of bivalent Ae cations with monovalent alkali metals, K, Cs, etc. For example, K for Sr substitution of 45 at.% in $Sr_{1-x}K_xFe_2As_2$ gives the maximum T_c [14–19].

(c) Undoped compounds, KFe_2As_2 and $CsFe_2As_2$, made of $[Fe_2As_2]^{-1}$ layers separated by monovalent ions are superconducting, with low T_cs of 3.8 and 2.6 K.

(d) Undoped LiFeAs made of $[FeAs]^{-1}$ layers is a superconductor with $T_c = 18$ K [20].

(e) Undoped non-superconducting $AeFe_2As_2$ (Ae = Ca, Sr, Ba) compounds, made of $[Fe_2As_2]^{-1}$ layers, become superconductors under pressure [21–23].

2. The internal pressure: misfit strain

There is clear evidence that the high-T_c superconductivity in these multilayer systems occurs by tuning the chemical potential at a particular point of the electronic structure. The chemical potential can be tuned by different methods: (1) by changing the charge density in the active layers via the control of the charge transfer, from or to the active layers; this is achieved by substitution of ions in the spacer layers, with others having a different ionic charge; (2) by changing the lattice parameters of the heterostructure at the atomic limit by external pressure or internal chemical pressure.

The chemical pressure in these superlattices is due to the lattice mismatch, called misfit strain, between the alternated layers. The misfit strain could control the bond distance in the active superconducting layers and could induce a disorder, due to the formation of dislocations [24], lattice stripes, and incommensurate lattice modulations [25]. The misfit strain is usually defined as $\eta = (a_1 - a_2)/a$, where a_1 and a_2 are the unit cell parameters of the ideal first and second layers, respectively, when they are well separated, and $a = (a_1 + a_2)/2$.

The first layers of the superlattice exhibit a compressive $\varepsilon_{1c} = (a_1 - a)/a$ microstrain and the second layers a tensile $\varepsilon_{2t} = (a - a_2)/a$ microstrain. The average strain is zero in the superlattice, $\varepsilon = \varepsilon_{1c} = \varepsilon_{2t}$, and the lattice parameter of the superlattice is close to $a = (a_1 + a_2)/2$. Therefore the internal chemical pressure (misfit strain or mismatch chemical pressure) acting on the active layers in the superlattice can be obtained by measuring the lattice parameter of the superlattice, a, and the unrelaxed ideal lattice parameter of only one of the two layers, in fact $\eta = \varepsilon_{1c} + \varepsilon_{2t} = 2\varepsilon$.

The chemical pressure is changed in cuprates [26–30] and in diborides [13] by chemical substitution of ions with different ionic radii in the spacer layers (in manganites $\eta = 1 - t$, where t is the tolerance factor). The chemical pressure acts as a complex anisotropic stress tensor that produces a compressive microstrain of the bcc CuO_2 layer in cuprates [26–30] and a tensile microstrain of the graphene-like B layer in magnesium diboride [13]. In cuprates, by increasing the chemical pressure at constant doping, $\delta = 1/8$, a structural phase transition from low-temperature orthorhombic, LTO, to low-temperature tetragonal, LTT, occurs at a critical compressive misfit strain of 8% [27–30]. In the proximity of this structural phase transition a nanoscale phase separation [31] in the presence of quenched disorder and a commensurate–incommensurate transition [32] for mobile dopants have been observed. Also the lattice structure of magnesium diborides is on the verge of a catastrophe [13]. The misfit strain of the FeAs quasi-2D layers can be easily measured. In fact these layers are made of edge sharing $FeAs_4$ tetrahedral units where the FeAs bond length remains constant, $R_0 = 240$ pm, therefore the chemical mismatch pressure induces only a rotation of the bonds pushing the As–Fe–As bond out of the ideal value of the tetrahedral angle 109°28′ [8] where the ideal lattice parameter of the orthorhombic lattice is $a_o = \sqrt{2}a_T = 542.7$ pm. The misfit strain, measuring the chemical internal pressure, is therefore given by $\eta = 2(a_o/542.7 - 1) = 2(a_T/390.8 - 1)$.

In this work have measured the misfit strain of the undoped parent compounds of the FeAs-based superconductor RFeAsO systems by powder x-ray diffraction. The ROFeAs (R = La, Pr, Nd and Sm) powder samples were synthesized in Beijing, as described elsewhere [3–5]. The x-ray diffraction patterns were recorded at the x-ray diffraction beam line at the Elettra synchrotron radiation facility in Trieste. The results of the measurement of the misfit strain as a function of the ionic radius in the spacer layers are shown in figure 1 (panel (a)). The lattice parameters of the '122' systems shown in panel (b) are taken from the literature. It is clear from the data that the FeAs layers in all undoped '1111' and '122' systems suffer a tensile misfit strain. The chemical doping giving the superconducting phase doping not only changes the charge transfer from the spacer to the FeAs layers but also the misfit strain. The tensile misfit strain is reduced in '1111', and in '122' samples at optimum doping the misfit strain is close to zero [8]. A large tensile misfit strain promotes the low-temperature charge and spin ordering phase that competes with superconductivity, and high-T_c superconductivity prevails where the misfit strain goes to zero. A compressive misfit strain shows up in low-temperature superconductors made of the '122' structure with intercalated monovalent alkali metal ions.

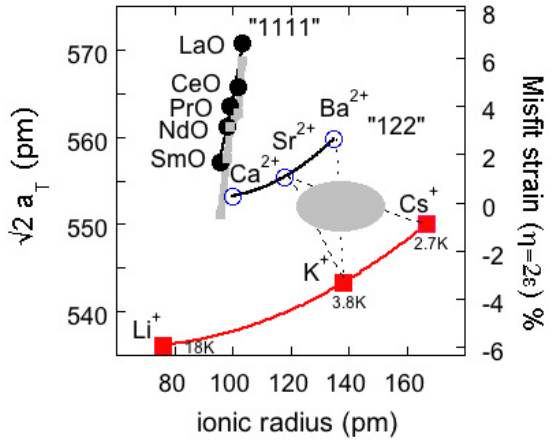

Figure 1. The values of the lattice parameter $a_o = \sqrt{2}a_T$, where a_T is the a axis of the tetrahedral high-temperature structure of the FeAs-based stoichiometric parent compounds with the '1111' structure (filled circles) as a function of the ionic radius of the ions in the spacer layers measured by powder x-ray diffraction. The a_o axes measuring twice the Fe–Fe distance for the system with the '122' structure having divalent alkaline-earth ions (open circles) or monovalent alkali ions (filled squares) taken from the literature are reported. The chemical internal pressure or misfit strain on the left axis is given by $\eta = 2(a_o/542.17 - 1)$. The superconducting samples at optimum doping have a microstrain close to zero [8]. The high-T_c superconductivity shows up in doped systems located in the grey regions.

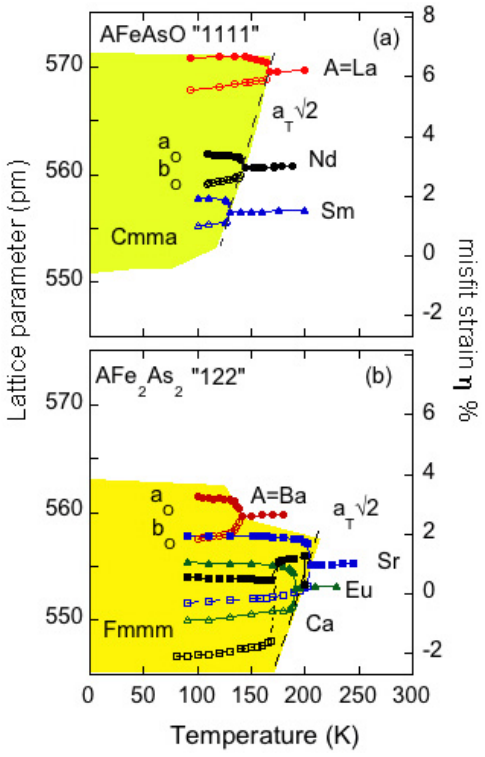

Figure 2. The structural parameters a_o and b_o of the orthorhombic structure and $\sqrt{2}a_T$ of the tetragonal structure for the stoichiometric undoped parent compounds of the FeAs-based superconductors as a function of temperature, showing the structural phase transition from the high-temperature tetragonal phase to the low-temperature orthorhombic phase.

3. The tetragonal to orthorhombic structural transition

It is known that by tuning the chemical potential at an electronic topological transition (ETT) the electron gas shows a 2.5 Lifshitz electronic topological transition; the compressibility of the electron gas becomes negative, and therefore the system has a tendency toward a first-order electronic phase separation; and finally the system can undergo a structural phase transition or a magnetic phase transition due to the freezing of a charge density wave (CDW) or a spin density wave (SDW) with the nesting vector connecting different portions of the Fermi surface. In this condition the system is near a magnetic, charge, orbital and lattice instability associated with the appearance of SDWs, CDWs, structural phase transitions and mesoscopic phase separation (MePhS). The electronic instabilities at the ETTs have been widely studied in the case of one-dimensional (1D) and two-dimensional (2D) single-band systems with the formation of 1D CDW and 2D CDW insulating phases respectively. A poorly studied case of interest here for the high-T_c cuprate superconductors is the onset of a 1D CDW (and/or SDW) in a 2D electronic system; in fact the 1D CDW opens only partial gaps in the 2D Fermi surface, forming a striped electronic matter with pseudogaps, multiple quasi-1D subbands and 2D bands that below T_c give a multigap superconducting phase. The second complex case that has been object of very few investigations is the one relevant for MgB_2 and FeAs-based superconductors: the case of multiband systems, where the Fermi level is crossing several types of ETTs only in one of the bands.

All undoped parent compounds of the FeAs-based layered superconductors are multiband systems [33–42], where the chemical potential is self-tuned to a particular point such that the system shows a lattice instability driven by a Fermi surface nesting wavevector. In fact all undoped parent compounds show a similar tetragonal to orthorhombic transition occurring at low temperature, T_s, detected by high-resolution x-ray diffraction. We show in figure 2 the splitting of the a axis in stoichiometric ROFeAs '1111' systems at the structural transition from a tetragonal (space group $P4/nmm$) to orthorhombic (space group $Cmma$) structure at low temperature, observed in the systems with R = La, Nd, and Sm in agreement with previous works [43–50]. In figure 2 we report the XRD results for AFe_2As_2 '122' systems that show a similar structural transition from tetragonal $ThCr_2Si_2$-type, space group $I4/mmm$, to orthorhombic $Fmmm$ space group [51–55]. In the orthorhombic phase a static stripe magnetic phase has been found. The structural transition takes place in a range of about 2 K in '122' systems, and it has been interpreted as a first-order transition [55] since it shows hysteretic behaviour, but it has been also identified as a second-order transition from the investigation of the scaling of the order parameter [52]. The structural transition in the '1111' systems shows a continuous character over a large temperature range above and below the critical temperature [45, 46]. This structural transition is therefore

similar to the martensitic transition in alloys [56] and superconducting A15 compounds [57].

There is a strong coupling between magnetic and structural order parameters [58, 59]. The spin ordering below the critical temperature T_s is driven by the low-temperature orthorhombic phase and it shows a striped phase with the antiferromagnetic coupling in the direction of the long Fe–Fe bond (the orthorhombic a_o axis) and ferromagnetic coupling in the direction of the short Fe–Fe bond (the orthorhombic b_o axis). Therefore this striped magnetic phase shows that the ordering of spin is coupled with the ordering of a lattice distortion (Fe–Fe short and long bonds) that is similar to the striped phase at 1/8 doping in cuprates associated with ordering of long and short Cu–O bonds [30]. The results in figure 2 clearly show that the critical temperature T_c of the structural phase transition decreases with decreasing the tensile strain due to lattice misfit. It is possible to see that the $BaFe_2As_2$ case shows an anomalous behaviour.

4. The mesoscopic phase separation at the orthorhombic to tetragonal structural transition

The superconducting phase is observed to emerge from the non-superconducting magnetically ordered phase through appropriate doping of the charge reservoir spacer blocks. Due to the complex Fermi surface of the FeAs superconductors [41, 42] the effects of doping on the electronic and superconducting properties are not clear. The FeAs-based materials are quite different from cuprates since the parent compounds are metallic systems and not Mott insulators. There is in contrast a strong analogy with the high-T_c cuprates if one assumes that the parent compound of all cuprates superconductors is the striped phase, at 1/8 doping and 8% misfit strain. In fact several authors [30, 60–66] have proposed that the relevant quantum critical point for high-T_c superconductivity in cuprates is where the superconducting phase competes with the striped phase, at 1/8 doping and 8% misfit strain.

Band structure calculations reveal that the Fermi surface includes electron as well as hole pockets. For superconducting $K_{1-x}Sr_xFe_2As_2$ near the optimal chemical substitution ($x_{optimum}$) the Hall coefficient was found to be positive, hinting that the majority carriers are holes [67]. However, with complete substitution, $x = 1$, the negative Hall coefficient of Sr-122 indicates that electrons dominate the transport properties [68]; therefore, increasing x introduces more electrons into the Fe_2As_2 layer and the chemical potential crosses the ETTs of these multiband system. These results are consistent with recent measurements [68, 69] and band structure calculations [34]. We show in figure 3 the variation of T_c and T_s for $K_{1-x}Sr_xFe_2As_2$ from [70]. Furthermore, the pressure-induced superconductivity in the non-superconducting compounds $AeFe_2As_2$ (Ae = Ca, Sr, Ba) indicates the role of the lattice in tuning the chemical potential. Therefore the high-T_c phase can be reached by varying the lattice parameters (modified by the external pressure or internal pressure) and the carrier densities in the Fe_2As_2 layers. The pressure experiments in $K_{1-x}Ba_xFe_2As_2$ [23] and

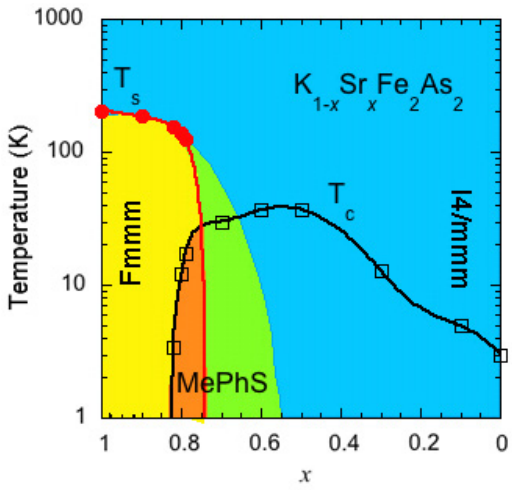

Figure 3. The superconducting critical temperature T_c (open squares) for $K_{1-x}Sr_xFe_2As_2$ [70] as a function of Sr content, x. The critical temperature of the structural phase transition T_s (filled circles) decreases to zero by decreasing x, reaching a quantum critical point where the superconducting order in the tetragonal lattice competes with the magnetic striped phase in the orthorhombic lattice. The superconducting critical temperature reaches a maximum at $x = 0.55$ and then decreases. A mesoscopic phase separation (MePhS) is clearly observed in the range $0.8 < x < 0.5$ with coexisting tetragonal and orthorhombic nanoclusters.

$K_{1-x}Sr_xFe_2As_2$ [70] show that the critical temperature is a function of both lattice parameters and charge density in the active FeAs layers, and the maximum T_c occurs along a line of points of charge density and pressure.

There is now a large agreement that by using pressure, internal pressure (as is shown in figure 2), and doping (as is shown in figure 3), it is possible to decrease the temperature T_s of the structural and magnetic phase transition towards zero. For the case $K_{1-x}Sr_xFe_2As_2$, the system reaches a quantum critical point, where $T_s = 0$ K, at a critical internal pressure (misfit strain), a critical charge density in the Fe 3d bands, and a critical disorder. The superconducting critical temperature reaches a maximum at $x = 0.55$, as is shown in figure 3.

The system shows a mesoscopic phase separation (MePhS) of orthorhombic striped magnetic clusters and tetragonal superconducting clusters in the proximity of the quantum critical point for the structural phase transition in figure 3. We show a pictorial view of this MePhS in figure 4 in a first high-doping regime, where the average structure is the tetragonal lattice, and in a second low-doping regime, where the average structure is the orthorhombic lattice. In the orthorhombic clusters the charges can move only along the stripes in the b-direction and are localized by the magnetic interaction in the a-direction. Therefore the first superconducting regime can be called a case of nematic electronic phase of itinerant fluctuating striped bubbles.

Therefore in the proximity of the zero temperature transition from the average orthorhombic phase to the tetragonal phase (a quantum phase transition driven by charge density, chemical pressure, or pressure) there should be a Fermi surface that fluctuates in space and time between a 2D

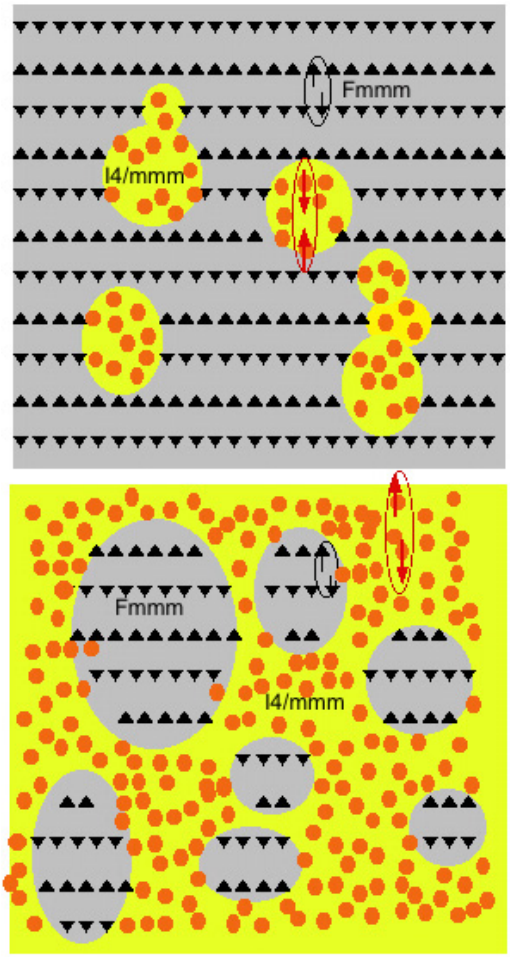

Figure 4. Pictorial view of the fluctuating mesoscopic phase separation regime (MePhS) (a) (upper panel) in the orthorhombic phase in the proximity of the structural phase transition from the orthorhombic ($Fmmm$) to the tetragonal ($I4/mmm$) structure where a fluctuation of nanoscale bubbles of metallic phase with a 2D Fermi surface (filled circles) coexists with the matrix of striped magnetic matter (triangles) with a quasi-1D Fermi surface; and the scenario (b) (lower panel) in the tetragonal phase in the proximity of the structural phase transition from tetragonal ($I4/mmm$) to orthorhombic ($Fmmm$) structure where a fluctuation of nanoscale bubbles of striped magnetic matter (triangles) with a quasi-1D Fermi surface coexists with the matrix of metallic phase (filled circles) with a 2D Fermi surface. The Feshbach resonance in the interband pairing is described as the exchange of a pair of electrons in the 2D Fermi surface in the tetragonal phase with a pair of electrons in the striped matter with a quasi-1D Fermi surface.

topology in the tetragonal clusters and a 1D topology in the orthorhombic clusters.

5. The Feshbach resonance in a fluctuating striped phase

We propose for FeAs-based superconductors a pairing mechanism, called the 'shape resonance' or 'Feshbach resonance' scenario, that has been proposed for the cuprates [64–66] and diborides [71]; it is similar to the pairing mechanism in ultracold gases called 'Feshbach resonance'. The key interband pairing process is a Kondo exchange-like interaction between *first pairs* (with spin up and spin down) in a first Fermi surface portion and the *second pairs* in a second Fermi surface portion. The two sets of electronic states are located in different spatial locations.

The 'shape resonances' have been described by Feshbach in elastic scattering cross-section for the processes of neutron capture and nuclear fission [72] in the cloudy crystal ball model of nuclear reactions. This scattering theory deals with configuration interaction in multichannel processes involving states with *different spatial locations.* Therefore these resonances can be called Feshbach shape resonances. These resonances are a clear well established manifestation of the non-locality of quantum mechanics and appear in many fields of atomic physics [73] and chemistry such as molecular association and dissociation processes [74, 75]. Feshbach resonances for molecular association and dissociation have been proposed for the manipulation of the interatomic interaction in ultracold atomic gases. In fact the interparticle interaction shows resonances tuning the chemical potential of the atomic gas around the energy of a discrete level of a biatomic molecule controlled by an external magnetic field [76]. This quantum phenomenon has been used to achieve Bose–Einstein condensation (BEC) in the dilute bosonic gases of alkali atoms [77] and to get a BCS-like condensate in fermionic ultracold gases with large values of T_c/T_F [78]. The process for increasing T_c by a Feshbach resonance was first proposed by Blatt and Thompson [79–81] in 1963 for a superconducting thin film, and it was called by Blatt 'shape resonance'. Blatt described the shape resonance in a superconducting thin film of thickness L where the chemical potential crosses the bottom E_n of the nth subband of the film, a quantum well, characterized by $k_z = n\pi/L$ with $n > 1$. Therefore it occurs where the chemical potential E_F is tuned near the critical energy $E_F = E_n$ for a 2.5 Lifshitz electronic topological transition (ETT) [82] at a band edge. At this ETT a small Fermi surface of a second subband disappears while the large 2D Fermi surface of a first subband shows minor variations. In the 'clean limit' the single electrons cannot be scattered from the nth to the $(n-1)$th subband and vice versa because of disparity and negligible spatial overlap, but configuration interaction between pairs in different subbands is possible in an energy window around $E_F = E_n$. Therefore the Feshbach shape resonance occurs by tuning the Lifshitz parameter $z = E_F - E_n$ around $z = 0$. In the Blatt proposal z is tuned by changing the film thickness. The prediction of Blatt and Thompson of the oscillatory behaviour of T_c as a function of film thickness L has been recently confirmed experimentally for a superconducting film [83], although phase fluctuations due to the electron confinement in the two dimensions is expected to reduce the critical temperature.

We have proposed to increase T_c via a Feshbach or shape resonance not in a single layer but in a multilayer (or superlattice) made of superconducting layers intercalated by spacer layers [84–93] in the proximity of a quantum critical point. This proposal was advanced following the experimental evidence that, in cuprates made of a superlattices

of CuO_2 layers intercalated by rocksalt spacer layers, the CuO_2 plane shows nanoscale striped lattice fluctuations detected by EXAFS with a timescale of 10^{-15} s [84], that is a signature for the proximity to the structural critical point for the LTO to LTT phase transition.

To describe the basic physics of superconductivity in these systems we have to overcome the approximations of the standard BCS theory for homogeneous systems considering an effective single band and focus on multiband superconductivity and anisotropic pairing mechanisms. In fact the standard BCS approximation assumes the Fermi energy at an infinite distance from the top or the bottom of the conduction band, and the pairing mechanism is not electronic state dependent (*the isotropic approximation*). The BCS wavefunction of the superconducting ground state is constructed by configuration interaction of all electron pairs ($+k$ with spin up, and $-k$ with spin down) on the Fermi surface in an energy window that is the energy cut-off of the interaction,

$$|\Psi_{\rm BCS}\rangle = \prod_k (u_k + v_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow})|0\rangle, \tag{1}$$

where $|0\rangle$ is the vacuum state, and $c^\dagger_{k\uparrow}$ is the creation operator for an electron with momentum k and spin up.

In anisotropic superconductivity one has to consider configuration interaction between pairs, in an energy window ΔE around the Fermi level, in different locations of the k-space with a different pairing strength, that gives a k-space dependent superfluid order parameter i.e., a k-dependent superconducting gap. A particular case of anisotropic superconductivity is multiband superconductivity, where the order parameter and the excitation gap are mainly different in different bands. The theory of two-band superconductivity, including the configuration interaction of pairs of opposite spin and momentum in the a-band and b-band, the many-body wavefunction is given by

$$|\Psi_{\rm Kondo}\rangle = \prod_k (u_k + v_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}) \prod_k (x_k + y_k b^\dagger_{k\uparrow} b^\dagger_{-k\downarrow})|0\rangle. \tag{2}$$

The element corresponding to the transfer of a pair from the a-band to the b-band or vice versa appears with the negative sign in the expression of the energy. This gain of energy is the origin of the increase of the transition temperature driven by interband pairing. Two-band superconductivity has been proposed for metallic elements and alloys [94–119], for doped cuprate perovskites [120–152], for magnesium diboride [71, 153–178] and for few other materials such as Nb-doped $SrTiO_3$ [179], Sr_2RuO_4 [180, 181] YNi_2B_2C, $LuNi_2B_2C$ [182] and $NbSe_2$ [183] and superlattices of carbon nanotubes [184].

The multiband superconductivity shows up only in the 'clean limit', where the single-electron mean free path for the interband impurity scattering satisfies the condition $l > h v_{\rm F}/\Delta_{\rm av}$, where $v_{\rm F}$ is the Fermi velocity and $\Delta_{\rm av}$ is the average superconducting gap. Therefore the criterion that the mean free path should be larger than the superconducting coherence length must be met. This is a very strict condition that implies also that the impurity interband scattering rate γ_{ab} should be very small: $\gamma_{ab} \ll (1/2)(K_{\rm B}/\hbar)T_{\rm c}$. Therefore most of the metals are in the 'dirty limit', where the interband impurity scattering mixes the electron wavefunctions of electrons on different spots on the bare Fermi surfaces and it reduces the system to an effective single Fermi surface.

The 'interchannel pairing' or 'interband pairing' that transfers a pair from the a-band to the b-band and vice versa in the multiband superconductivity theory is expressed by the off-diagonal element

$$\sum_{k,k'} J(k,k')(a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow} b^\dagger_{-k\downarrow} b^\dagger_{k\uparrow}), \tag{3}$$

where $a^\dagger$ and $b^\dagger$ are creation operators of electrons in the a-band and b-band, respectively, and $J(k,k')$ is an exchange-like integral. This interband pairing interaction may be repulsive as it was first noticed by Kondo [97]. Therefore it is a non-BCS pairing process since in the BCS theory an attractive interaction is required for the formation of Cooper pairs. Another characteristic feature of multiband superconductivity is that the order parameter shows a sign reversal in the case of a repulsive interband pairing interaction.

The non-BCS nature of the interband pairing process is indicated also by the fact that, when it is dominant, the isotope effect vanishes even if the intra-band attractive interaction in each band is due to the electron–phonon coupling. Moreover, the effective repulsive Coulomb pseudopotential in the Migdal–Eliashberg theory is expected to decrease (so that the effective coupling strength increases) where the interband pairing is dominant.

6. The Feshbach resonance at the topological electronic transitions in a simple multiband system

FeAs-based superconductors, like all other superconductors, are multiband systems where the Fermi level in one of the bands is close to a band edge, a hole–electron van Hove singularity, a 2D to 3D electronic topological transition, or a 1D to 2D electronic topological transition. Therefore, here there is a breakdown of both 'the infinite Fermi energy approximation' and 'the single-band approximation' for the standard BCS for low-temperature superconductors. This is a common feature for all high-$T_{\rm c}$ superconductors known so far. Therefore the superconducting order parameter is expected to be dependent on the details of the electronic structure and k-dependent, as is observed in these anisotropic multiband systems. The anisotropic multiband scenario introduces new possible terms in the pairing process where the superconducting condensate is formed taking advantage of the Coulomb interactions between the fermions itself, via exchange of spin fluctuations between nested portions of the Fermi surfaces or acoustic plasmons between different portions of the Fermi surfaces. The Feshbach resonance due to exchange of pairs between different portions of the Fermi surfaces could be the key mechanism for making a quantum condensate that avoids temperature decoherence effects. In fact in multiband superconductors there are some special conditions where the exchange-like interband pairing could show Feshbach resonance.

Here we discuss a particular case of multiband superconductivity that grabs some key feature of Feshbach resonances in FeAs-based superconductors. We consider a toy electronic structure model: the case of a superlattice of quantum wires that simulates the electronic structure of a striped metallic and magnetic phase. Here the charge carriers in the superconducting layer move as free charges in the x-direction (the short b_o-axis in the $FeAs_{4/4}$ 2D lattice) but they have to overcome a periodic potential barrier $V(x, y)$, with period λ_p, amplitude V_b and width W along the y-direction (the long a_o-axis in the $FeAs_{4/4}$ 2D lattice) constant in the x-direction, expressed for $x =$ constant as

$$V(y) = -V_b\theta\left(\frac{L}{2} - \tilde{y}\right) \qquad \text{where } \tilde{y} = y - q\lambda_p - \frac{\lambda_p}{2} \quad (4)$$

where q is the integer part of y/λ_p.

The solution of the Schrödinger equation for this system

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x, y) + V(x, y)\,\psi(x, y) = E\psi(x, y),$$

where

$$\psi_{n,k_x,k_y}(x, y) = e^{ik_x x}e^{ik_y q\lambda_p}\psi_{n,k_y}(y), \qquad (5)$$

in the stripe is given by

$$\psi_{n,k_y}(y) = \alpha\, e^{ik_w\tilde{y}} + \beta e^{-ik_w\tilde{y}} \qquad \text{for } |\tilde{y}| < L/2$$

$$k_w = \sqrt{2m_w(E_n(k_y) + V_b)/\hbar^2},$$

and in the barrier is given by

$$\psi_{n,k_y}(y) = \gamma e^{ik_b\tilde{y}} + \delta e^{-ik_b\tilde{y}} \qquad \text{for } |\tilde{y}| \geqslant L/2$$

$$k_b = \sqrt{2m_b E_n(k_y)/\hbar^2}.$$

The coefficients α, β, γ, and δ are obtained by imposing the Bloch conditions with periodicity λ_p, the continuity conditions of the wavefunction and its derivative at $L/2$, and finally by normalization in the surface unit. The solution of the eigenvalue equation for E gives the electronic energy dispersion for the n subbands with energy $\varepsilon_n(k_x, k_y) = \varepsilon(k_x) + E_n(k_y)$, where $\varepsilon(k_x) = (\hbar^2/2m)k_x^2$ is the free electron energy dispersion in the x-direction and $E_n(k_y)$ is the dispersion in the y-direction.

There are N_b solutions for $E_n(k_y)$, with $1 \leqslant n \leqslant N_b$, for each k_y in the Brillouin zone of the superlattice, giving a dispersion in the y-direction of the N_b subbands with $k_x = 0$.

By changing the charge density it is possible to cross the ETT where the Fermi surface topology of the second subband changes form 2D to 1D as shown in figure 5. The partial density of states (DOS) of the nth subband gives a step-like increase of the total DOS when the chemical potential reaches the bottom of the subband $n = 2$ at energy E_0 where an ETT (appearing of a new Fermi surface spot) occurs as it is shown in figure 6. At the change from 2D to 1D topology a peak in the DOS is observed, as is shown in figure 6.

The superlattice induces a relevant k-dependent interband pairing interaction $V_{n,n'}(k, k')$, that is, the exchange-like non-BCS interband pairing interaction. The interband interaction

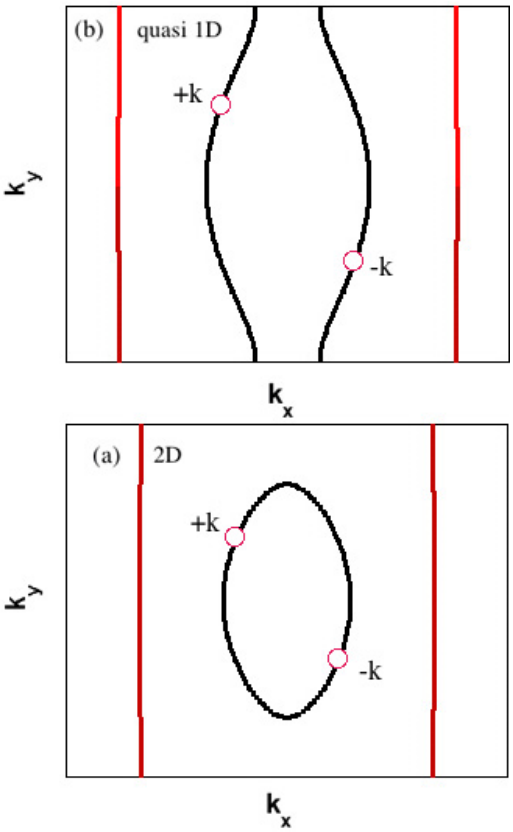

Figure 5. (a) The Fermi surface above the bottom of the second subband made of a first 1D subband (vertical red lines) and a second 2D subband (black circle). By changing the pressure, charge density, or misfit strain it is possible to cross an ETT where the DOS shows a sharp peak (see the figure) and the second subband changes its topology from a 2D topology (panel (a)) to a 1D topology (panel (b)). A different type of ETT (appearance of a new Fermi surface spot) occurs where the second subband disappears since it is not crossing the Fermi level.

is controlled by the details of the *quantum superposition of states* between the wavefunctions of the pairing electrons in the different subbands of the superlattice.

$$V_{n,n'}(k, k') = V^o_{n,k_y;n',k'_y}\theta(\hbar\omega_0 - |\varepsilon_n(k) - \mu|) \times \theta(\hbar\omega_0 - |\varepsilon_{n'}(k') - \mu|), \qquad (6)$$

where $k = (k_x, k_y)$ and

$$V^o_{n,k_y;n',k'_y} = -J\int_S dx\, dy\, \psi_{n,-k}(x, y)\psi_{n',-k'}(x, y) \times \psi_{n,k}(x, y)\psi_{n',k'}(x, y)$$
$$= -J\int_S dx\, dy\, |\psi_{n,k}(x, y)|^2|\psi_{n',k'}(x, y)|^2,$$

where n and n' are the subband indices. k_x ($k_{x'}$) is the component of the wavevector in the wire direction (or longitudinal direction), k_y ($k_{y'}$) is the superlattice wavevector (in the transverse direction) of the initial (final) state in the pairing process, and μ is the chemical potential.

In the separable kernel approximation, the gap parameter has the same energy cut-off $\hbar\omega_o$ as the interaction. Therefore it takes the values $\Delta_n(k_y)$ around the Fermi surface in a range

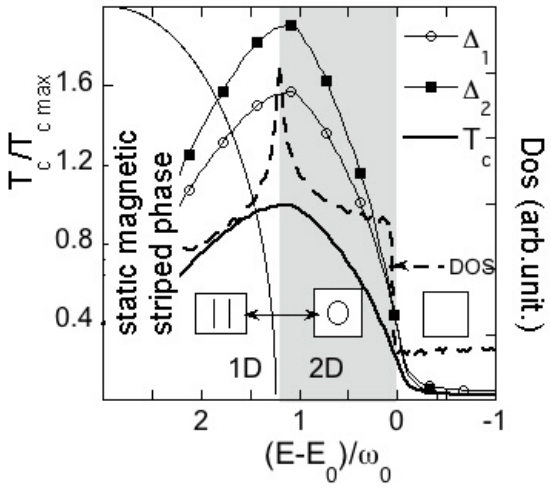

Figure 6. The total density of states (DOS) for a superlattice of quantum wires near the bottom of the second subband (dashed line) for the case where the ratio between the pairing energy cut-off (ω_o) and the transversal energy dispersion (D) $\omega_o/D = 0.86$. The critical temperature T_c, and the superconducting gaps in the first, Δ_1, and second, Δ_2, subband for a superlattice of quantum wires are normalized to the maximum T_c and plotted as a function of the ratio of the Fermi level minus the band edge to the energy cut-off ω_o.

$\hbar\omega_o$ depending on the subband index and the superlattice wavevector k_y.

The self-consistent equation for the ground state energy gap $\Delta_n(k_y)$ is

$$\Delta_n(\mu, k_y) = -\frac{1}{2N}\sum_{n'k'_yk'_x}\frac{V_{n,n'}(k,k')\Delta_{n'}(k'_y)}{\sqrt{(E_{n'}(k'_y)+\varepsilon_{k'_x}-\mu)^2+\Delta^2_{n'}(k'_y)}}, \tag{7}$$

where N is the total number of wavevectors. Solving this equation iteratively gives the anisotropic gaps, which are dependent on the subband index and weakly dependent on the superlattice wavevector k_y. The structure in the interaction gives different values for the gaps Δ_n, giving a system with an anisotropic gaps in the different segments of the Fermi surface.

The superconducting gaps in the second, Δ_2, and first, Δ_1, subband in a superlattice of quantum wires are shown in figure 5.

The increase of the gap Δ_1 is driven only by the Feshbach resonance in the interband pairing since the partial DOS of the first subband does not have peaks.

The critical temperature T_c of the superconducting transition can be calculated by an iterative method:

$$\Delta_n(k) = -\frac{1}{N}\sum_{n',k'}V_{nn'}(k,k')\frac{\mathrm{tgh}\left(\frac{\xi_{n'}(k')}{2T_c}\right)}{2\xi_{n'}(k')}\Delta_{n'}(k'), \tag{8}$$

where $\xi_n(k) = \varepsilon_n(k) - \mu$.

The interband pairing term enhances T_c by tuning the chemical potential in an energy window around the Van Hove singularities, 'z' $= 0$, associated with a change of the topology of the Fermi surface from one dimension to two dimensions of the second subband of the superlattice.

The critical temperature T_c and the superconducting gap in the first 1D subband and in the second 2D subband are plotted in figure 6.

The chemical potential is normalized to the cut-off energy ω_0 that is the energy window around the Fermi energy where the electrons pairs are that contribute to the formation of the macroscopic quantum condensate.

In the FeAs-based superconductors near the quantum critical point for the Lifshitz ETT transition, the Van Hove feature in the electronic energy spectrum associated with a change of Fermi surface topology from one dimension to two dimensions fluctuates around the Fermi level due to quantum fluctuations. The amplitude of the energy fluctuations controls the energy window where the electron pairs that contribute to the formation of the macroscopic quantum condensate wavefunction are located. Therefore the cut-off energy ω_o for the pairing is related to the quantum fluctuations.

We think that our calculations in figure 6 reproduce the basic experimental results of figure 3 and provide a qualitative understanding of the multigap superconductivity in FeAs superconductors. In the low-temperature orthorhombic magnetic striped phase the electronic structure should be similar to the one described here where the Fermi level is in the pseudogap where all bands have a quasi-1D character. In this regime the static magnetism prevails on the superconducting order. Doping the system changes the position of the chemical potential and increases the randomness of the system up to a critical point where the first-order phase transition becomes a continuous first transition with a fluctuating nanoscale phase separation, as is shown in figure 3. The superconducting phase emerges where at least one subband reaches the 1D–2D electronic topological transition. Considering the Fermi surface of superconducting ($T_c = 32$ K) $Ba_{1-x}K_xFe_2As_2$ reported recently [42], we identify this subband with the '*inner* Γ *barel*'. The '*inner* Γ *barel*' is simulated by the *second subband* in our toy theoretical model described above. In fact the '*inner* Γ *barel* is clearly a small 2D Fermi level close to the band edge and it shows a large superconducting gap of 9 meV and $2\Delta/K_BT_c = 6.8$. The '*outer* Γ *barel*' is simulated by the first subband where the Fermi level is far from the band edge, the superconducting gap is smaller than 3 meV, and $2\Delta/K_BT_c < 3$.

The Fermi surface spots called '*inner* Γ *barel*' and the '*outer* Γ *barel*' should be strongly modulated by quantum fluctuations when the system is close to the zero temperature phase transition from the orthorhombic to the tetragonal structural phase and from the static striped phase to the superconducting phase shown in figure 3 with dynamical lattice and spin fluctuations as shown in figure 4.

Therefore we expect that, in our toy model, shown in figure 6, the onset of superconductivity appears by moving the chemical potential across the 1D–2D electronic topological transition. The superconducting critical temperature is different from zero where the chemical potential is in an energy range window around the ETT that in our model is the energy cut-off. In this regime the superconducting condensate is made of configuration interactions including both electrons pairs in a quasi-1D Fermi surface and pairs in the 2D Fermi surface of the second subband, as is shown in figure 5.

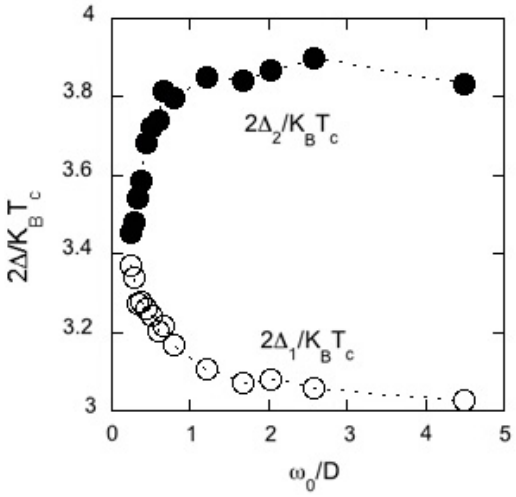

Figure 7. The ratio $2\Delta_1/K_BT_c$ and $2\Delta_2K_BT_c$ as a function of the ratio between the pairing energy cut-off (ω_o) and the transversal energy dispersion (D) between stripes. The largest deviation of the ratio $2K_BT_c$ from the expected BCS value 3.5 occurs for the large values of the ratio ω_o/D, while for small $\omega_o/D < 1$ it converges toward the BCS value.

Finally the chemical potential crosses the bottom of the band and the system goes into the Bose-like regime where all electrons in the second subband form the condensate. Therefore in this scenario where the Fermi level is tuned from the 1D to the 2D topology we expect that the system is in a regime of BCS–Bose crossover that is typical of cuprates and FeAs superconductors that follows the Uemura plot.

Finally, in figure 7 we report the variation of the ratio $2\Delta/K_BT_{c\,max}$ for the two gaps where the superconducting T_c is maximum. The experimental values of the ratio $2\Delta/K_BT_{c\,max}$ measured so far in FeAs superconductors are 6.8 and smaller than 3, showing a large deviation from the standard BCS value of 3.5. In our calculations we show that the larger deviation from the BCS value occurs where $\omega_0 < D$.

In conclusion, the new data discussed in this work point towards the Feshbach resonance of the exchange-like pairing at the ETT associated with the Fermi surface topology crossover as a possible scenario that grabs some key physics of the high superconductivity at the BCS to Bose crossover near a quantum critical point that is common between multiband cuprates [185], FeAs pnictides, diborides, and doped nanotubes [184]. We have provided evidence that the tuning of the chemical potential to the quantum critical point of the order–disorder transition of the stripes phase (analogous to the 1/8 stripe phase in cuprates) is also controlled by misfit strain beyond doping and disorder. The tuning drives the system at the verge of a catastrophe, i.e. near the stripes' order-to-disorder phase transition, having a critical temperature T_s going to zero. At this quantum critical point the quantum lattice and magnetic fluctuations promote the Feshbach resonance. The superconducting gaps are controlled by the interplay of the hopping between stripes and the quantum fluctuations. The recent detection of the multigaps by the ARPES experiment of Borisenko's group on the Fermi surface spots [42] is shown to support the Feshbach resonance scenario.

Acknowledgments

We thank the staff of the XRD beam line of the Elettra synchrotron radiation facility in Trieste and Naurang L Saini for help and discussions. We acknowledge financial support from European STREP project 517039 'Controlling Mesoscopic Phase Separation' (COMEPHS) (2005).

References

[1] Kamihara Y, Watanabe T, Hirano M and Hosono H 2008 *J. Am. Chem. Soc.* **130** 3296
[2] Takahashi H, Igawa K, Arii K, Kamihara Y, Hirano M and Hosono H 2008 *Nature* **453** 376
Wen H H, Mu G, Fang L, Yang H and Zhu X 2008 *Europhys. Lett.* **82** 17009
[3] Ren Z A *et al* 2008 *Europhys. Lett.* **82** 57002
[4] Ren Z A *et al* 2008 *Europhys. Lett.* **83** 17002
[5] Ren Z A, Yang J, Lu W, Yi W, Che G C, Dong X L, Sun L L and Zhao Z X 2008 *Mater. Sci. Innov.* **12** 1
[6] Chen X H, Wu T, Wu G, Liu R H, Chen H and Fang D F 2008 *Nature* **453** 761
Chen G F, Li Z, Wu D, Li G, Hu W Z, Dong J, Zheng P, Luo J L and Wang N L 2008 *Phys. Rev. Lett.* **100** 247002
[7] Chen G F, Li Z, Wu D, Dong J, Li G, Hu W Z, Zheng P, Luo J L and Wang N L 2008 *Chin. Phys. Lett.* **25** 2235
[8] Lee C-H, Ito T, Iyo A, Eisaki H, Kito H, Fernandez-Diaz M T, Kihou K, Matsuhata H, Braden M and Yamada K 2008 *J. Phys. Soc. Japan* **77** 083704
[9] Rotter M, Tegel M and Johrendt D 2008 *Phys. Rev. Lett.* **101** 107006
[10] Jaroszynski J *et al* 2008 arXiv:0806.1352
[11] Bednorz J G and Müller K A 1986 *Z. Phys.* B **64** 189–93
[12] Georg Bednorz J and Alex Müller K 1988 *Rev. Mod. Phys.* **60** 585–600
[13] Agrestini S, Di Castro D, Sansone M, Saini N L, Saccone A, De Negri S, Giovannini M, Colapietro M and Bianconi A 2001 *J. Phys.: Condens. Matter* **13** 11689–95
[14] Rozsa S and Schuster H-U 1981 *Z. Naturf.* B **36** 1668
[15] Czybulka A, Noak M and Schuster H-U 1992 *Z. Anorg. Allg. Chem.* **609** 122
[16] Rotter M, Tegel M, Johrendt D, Schellenberg I, Hermes W and Pottgen R 2008 *Phys. Rev.* B **78** 020503(R)
[17] Rotter M, Tegel M and Johrendt D 2008 *Phys. Rev. Lett.* **101** 107006
[18] Krellner C, Caroca-Canales N, Jesche A, Rosner H, Ormeci A and Geibel C 2008 *Phys. Rev.* B **78** 100504(R)
[19] Sasmal K, Lv B, Lorenz B, Guloy A, Chen F, Xue Y Y and Chu C W 2008 *Phys. Rev. Lett.* **101** 107007
[20] Tapp J H, Tang Z, Lv B, Sasmal K, Lorenz B, Chu P C W and Guloy A M 2008 *Phys. Rev.* B **78** 060505(R)
[21] Alireza P, Gillett J, Ko Y T C, Sebastian S E and Lonzarich G G 2008 arXiv:0807.1896
[22] Fukazawa H, Takeshita N, Yamazaki T, Kondo K, Hirayama K, Kohori Y, Miyazawa K, Kito H, Eisaki H and Iyo A 2008 arXiv:0808.0718
[23] Torikachvili M S, Bud'ko S L, Ni N and Canfield P C 2008 *Phys. Rev. Lett.* **101** 057006
[24] Bak P 1982 *Rep. Prog. Phys.* **45** 587
Fisher M E 1984 *J. Stat. Phys.* **34** 667
[25] Jain S C, Harker A H and Cowley R A 1997 *Phil. Mag.* A **75** 1461
Bruce A D and Cowley R A 1978 *J. Phys. C: Solid State Phys.* **11** 3609
Cowley R A and Bruce A D 1978 *J. Phys. C: Solid State Phys.* **11** 3577

[26] Di Castro D, Bianconi G, Colapietro M, Pifferi A, Saini N L, Agrestini S and Bianconi A 2000 *Eur. Phys. J.* B **18** 617
[27] Bianconi A, Saini N L, Agrestini S and Di Castro D 2000 *Physica* B **14** 3342
[28] Agrestini S, Saini N L, Bianconi G and Bianconi A 2003 *J. Phys. A: Math. Gen.* **36** 9133
[29] Bianconi A, Agrestini S, Bianconi G, Di Castro D and Saini N L 2001 *J. Alloys Compounds* **317/318** 537
[30] Fratini M, Poccia N and Bianconi A 2008 *J. Phys. Conf.: Ser.* **108** 012036
[31] Bianconi A, Di Castro D, Bianconi G, Pifferi A, Saini N L, Chou F C, Johnston D C and Colapietro M 2000 *Physica* C **341–348** 1719
[32] Kusmartsev F V, Di Castro D, Bianconi G and Bianconi A 2000 *Phys. Lett.* A **275** 118
[33] Lebègue S 2007 *Phys. Rev.* B **75** 035110
[34] Singh D J and Du M H 2008 *Phys. Rev. Lett.* **100** 237003
[35] Boeri L, Dolgov O V and Golubov A A 2008 *Phys. Rev. Lett.* **101** 024603
[36] Yin Z P, Lebègue S, Han M J, Neal B, Savrasov S Y and Pickett W E 2008 arXiv:0804.3355
[37] Dong J *et al* 2008 arXiv:0803.3426
Cvetkovic V and Tesanovich Z 2008 arXiv:0808.3742
[38] Korshunov M and Eremin I 2008 *Europhys. Lett.* **83** 67003
[39] Yldirim T 2008 *Phys. Rev. Lett.* **101** 057010
[40] Liu C *et al* 2008 arXiv:0806.2147
Lu D H *et al* 2008 arXiv:0807.2009
[41] Liu C *et al* 2008 arXiv:0806.3453
Zabolotnyy V B *et al* 2008 arXiv:0808.2454
[42] Evtushinsky D V *et al* 2008 arXiv:0809.4455
[43] De La Cruz C *et al* 2008 arXiv:0804.0795
[44] Nomura T, Kim S W, Kamihara Y, Hirano M, Sushko P V, Kato K, Takata M, Shluger A L and Hosono H 2008 arXiv:0804.3569
[45] McGuire A M *et al* 2008 arXiv:0806.3878
[46] Fratini M *et al* 2008 *Supercond. Sci. Technol.* **21** 092002
[47] Chen Y *et al* 2008 arXiv:0807.0662
[48] Margadonna S, Takabayashi Y, McDonald M T, Brunelli M, Wu G, Liu R H, Chen X H and Prassides K 2008 arXiv:0806.3962
[49] Zhao J *et al* 2008 arXiv:0806.2528
[50] Martinelli A, Palenzona A, Ferdeghini C, Putti M and Emerich E 2008 arXiv:0808.1024
[51] Huang Q, Qiu Y, Bao W, Green M A, Lynn J W, Gaasparovic Y C, Wu T, Wu G and Chen X H 2008 arXiv:0806.27776
[52] Tegel M, Rotter M, Weiss V, Schappacher F M, Pottgen R and Johrendt D 2008 arXiv:0806.4782
[53] Zhao J, Ratcliff W, Lynn J W, Chen G F, Luo J L, Wang N L, Hu J and Dai1 P 2008 arXiv:0807.1077
[54] Yan J-Q *et al* 2008 arXiv:0806.2711
[55] Ni N *et al* 2008 *Phys. Rev.* B **78** 014523
Goldman A I *et al* 2008 arXiv:0807.1525
Krellner C, Caroca-Canales N, Jesche A, Rosner H, Ormeci A and Geibel C 2008 arXiv:0806.1043
[56] Katsnelson M I, Naumov I I and Trefilov A V 1994 *Phase Transit.* **49** 143–91
[57] Shirane G and Axe J D 1971 *Phys. Rev. Lett.* **27** 1803
[58] Jesche A *et al* 2008 arXiv:0807.0632
[59] Chen Y, Lynn J W, Li J, Li G, Chen G F, Luo J L, Wang N L, Dai P, de la Cruz C and Mook H A 2008 arXiv:0807.0662
[60] Kivelson S A, Aeppli G and Emery V J 2001 *Proc. Natl Acad. Sci.* **98** 11903–7
[61] Kivelson S A 2006 *Nat. Mater.* **5** 343
[62] Mohottala H E *et al* 2006 *Nat. Mater.* **5** 377–82
[63] Aeppli G *et al* 1997 *Science* **278** 1432–5
[64] Bianconi A 1994 *Physica* C **235–240** 269–72
[65] Bianconi A 1994 *Solid State Commun.* **91** 1
Bianconi A and Missori M 1994 *Solid State Commun.* **91** 287
[66] Bianconi A and Missori M 1994 Phase separation in cuprate superconductors *Proc. Workshop (Cottbus, Sept. 1993)* ed E Sigmund and K A Müller (Berlin: Springer) pp 272–89
[67] Chen G F, Li Z, Li G, Hu W Z, Dong J, Zhang X D, Zheng P, Wang N L and Luo J L 2008 *Chin. Phys. Lett.* **25** 3403
[68] Yan J-Q *et al* 2008 *Phys. Rev.* B **78** 024516
[69] Liu H *et al* 2008 arXiv:0806.4806
[70] Gooch M, Lv B, Lorenz B, Guloy A M and Chu C-W 2008 arXiv:0809.2054
[71] Bianconi A, Di Castro D, Agrestini S, Campi G, Saini N L, Saccone A, De Negri S and Giovannini M 2001 *J. Phys.: Condens. Matter* **13** 7383–90
[72] Feshbach H, Porter C E and Weisskopf V F 1954 *Phys. Rev.* **96** 448
Feshbach H 1958 *Ann. Phys.* **5** 357
[73] Fano U 1935 *Nuovo Cimento* **128** 156 (arXiv:cond-mat/0502210)
Fano U 1961 *Phys. Rev.* **124** 1866
[74] Fano U 1986 *Atomic Collisions and Spectra* (Amsterdam: Elsevier)
[75] Bianconi A 2002 X-ray and inner shell processes *AIP Conf. Proc.* 13–8
[76] Tiesinga E, Verhaar B J and Stoof H T C 1993 *Phys. Rev.* A **47** 4114
[77] Inouye S *et al* 1998 *Nature* **392** 151
[78] Regal C A, Greiner M and Jin D S 2004 *Phys. Rev. Lett.* **92** 040403
[79] Blatt J M and Thompson C J 1963 *Phys. Rev. Lett.* **10** 332
[80] Thompson C J and Blatt J M 1963 *Phys. Lett.* **5** 6
[81] Blatt J M 1964 *Theory of Superconductivity* (New York: Academic) for shape resonances see 362 pp and 215 pp
[82] Lifshitz I M 1960 *Sov. Phys.—JEPT* **11** 1130
[83] Guo Y *et al* 2004 *Science* **306** 1915
[84] Bianconi A, Saini N L, Lanzara A, Missori M, Rossetti T, Oyanagi H, Yamaguchi H, Oka K and Ito T 1996 *Phys. Rev. Lett.* **76** 3412
[85] Bianconi A, Saini N L, Rossetti T, Lanzara A, Perali A and Missori M 1996 *Phys. Rev.* B **54** 12018
[86] Bianconi A, Valletta A, Perali A and Saini N L 1997 *Solid State Commun.* **102** 369
[87] Perali A, Valletta A, Bardelloni G, Bianconi A, Lanzara A and Saini N L 1997 *J. Supercond.* **10** 355
Perali A, Bianconi A, Lanzara A and Saini N L 1996 *Solid State Commun.* **100** 181
[88] Valletta A, Bardelloni G, Brunelli M, Lanzara A, Bianconi A and Saini N L 1997 *J. Supercond.* **10** 383
[89] Valletta A, Bianconi A, Perali A and Saini N L 1997 *Z. Phys.* B **104** 707
[90] Saini N L, Avila J, Bianconi A, Lanzara A, Asenzio M C, Tajima S, Gu G D and Koshizuka N 1997 *Phys. Rev. Lett.* **79** 3467
[91] Bianconi A, Valletta A, Perali A and Saini N L 1998 *Physica* C **296** 269
[92] Bianconi A 2000 *Int. J. Mod. Phys.* B **14** 3289
[93] Saini N L and Bianconi A 2000 *Int. J. Mod. Phys.* B **14** 3649
[94] Moskalenko V A 1959 *Phys. Met. Metallogr.* **8** 25
[95] Suhl H, Matthias B T and Walker L R 1959 *Phys. Rev. Lett.* **3** 552
Matthias B T 1962 *J. Phys. Soc. Japan* **17** (Suppl. B-I) 104
[96] Suffczynski M 1962 *Phys. Rev.* **128** 1538
[97] Kondo J 1963 *Prog. Theor. Phys.* **29** 1
[98] Garland J W Jr 1963 *Phys. Rev. Lett.* **11** 111
[99] Zavaritskii N V 1963 *Sov. Phys.—JEPT* **18** 1260
[100] Schrieffer J R 1964 *Theory of Superconductivity* (New York: Benjamin) (for interband pairing see p 300)
[101] Geballe T H 1964 *Rev. Mod. Phys.* **36** 134
Tilley D R 1964 *Proc. Phys. Soc.* **84** 573
[102] Geilikman B T 1965 *Sov. Phys.—JEPT* **21** 796
[103] Yu L and Gu B-Y 1965 *Acta Phys. Sin.* **21** 838

[104] Moskalenko V A 1966 *Sov. Phys.—JETP* **24** 780
Moskalenko V A and Palistrant M E 1966 *Sov. Phys.—JETP* **22** 536
[105] Vasudevan R and Sung C C 1966 *Phys. Rev.* **144** 237
[106] Leggett A J 1966 *Prog. Theor. Phys.* **36** 901
Leggett J 1966 *Prog. Theor. Phys.* **36** 931
[107] Geilikman B T, Zaitsev R O and Kresin V Z 1967 *Sov. Phys.—Solid State* **9** 642
[108] Kon L Z 1967 *Phys. Met. Metallogr.* 23
[109] Garland J W Jr 1967 *Phys. Rev.* **153** 460
[110] Wong V C and Sung C C 1967 *Phys. Rev. Lett.* **19** 1236
Sung C C and Wong V C 1967 *J. Phys. Chem. Solids* **28** 1933
Chow W S 1968 *Phys. Rev.* **172** 467
[111] Kusakabe T 1970 *Prog. Theor. Phys.* **43** 907
[112] Palistrant M E and Kolpajiu M K 1972 *Phys. Lett.* A **41** 123
[113] Aronov A G and Sonin E B 1973 *JEPT Lett.* **36** 556
[114] Kresin V Z 1973 *J. Low Temp. Phys.* **11** 519
[115] Geilikman B T and Kresin V Z 1974 *Kinetic and Non Stationary Phenomena in Superconductors* (New York: Wiley) p 34 and 80
[116] Entel P and Peter M 1975 *J. Low Temp. Phys.* **22** 613
Entel P, Klose W, Fischer O and Bongi G 1975 *Z. Phys.* B **21** 363
Entel P and Peter M 1976 *J. Low Temp. Phys.* **22** 613
[117] Butler H and Allen P B 1976 *Superconductivity in d and f-band Metals* ed D H Douglass (New York: Plenum)
Allen P B 1978 *Phys. Rev.* B **17** 3725
Allen P B and Mitrovic B 1982 *Solid State Physics* ed F Seitz, D Turnbull and H Ehrenreich (New York: Academic) p 1
[118] Vignale G and Singwi K S 1985 *Phys. Rev.* B **31** 2729
[119] Ranninger J and Robaszkiewicz S 1985 *Physica* B **53** 468
[120] Lee D H and Ihm J 1987 *Solid State Commun.* **62** 811
[121] Yamaij K and Abe A 1987 *J. Phys. Soc. Japan* **56** 4237
Yamaij K 1987 *Solid State Commun.* **64** 1157
Yamaji K 1990 *J. Phys. Soc. Japan* **59** 677
Yamaji K 1994 *Physica* C **222** 349
[122] Robaszkiewicz S, Micnas R and Ranninger J 1987 *Phys. Rev.* B **36** 180
[123] Kresin V Z and Wolf S A 1987 *Novel Superconductivity* ed S A Wolf and V Z Kresin (New York: Plenum)
Ashcroft N W 1987 *Novel Superconductivity* ed S A Wolf and V Z Kresin (New York: Plenum) p 301
Kresin V Z 1987 *Solid State Commun.* **63** 725
Kresin V Z and Wolf S A 1987 *Solid State Commun.* **63** 1141
Kresin V Z and Wolf S A 1990 *Physica* C **169** 476
Kresin V Z and Wolf S A 1990 *Phys. Rev.* B **41** 4278
Kresin V Z and Wolf S A 1992 *Phys. Rev.* B **46** 6458
Kresin V Z, Wolf S A and Deutscher G 1991 *J. Supercond.* **4** 409
Kresin V Z and Wolf S A 1995 *Phys. Rev.* B **51** 1229
Adrian S D, Wolf S A, Dolgov O V, Shulga S and Kresin V Z 1997 *Phys. Rev.* B **56** 7878
Kresin V Z and Wolf S A 1987 *J. Supercond.* **1** 143
[124] Ruvalds J 1987 *Phys. Rev.* B **35** 8869
[125] Xu J-h 1988 *Solid State Commun.* **65** 135
[126] Konsin P, Kristoffel N and Ord T 1988 *Phys. Lett.* A **129** 399
Kristoffel N, Konsin P and Ord T 1994 *Riv. Nuovo Cimento* **17** 1
Konsin P, Kristoffel N and Rubin P 1996 *Solid State Commun.* **97** 567
Kristoffel N, Konsin P and Rubin P 1997 *Phys. Status Solidi* b **203** 501
Kristoffel N, Konsin P and Rubin P 1998 *Phys. Status Solidi* b **208** 145
Kristoffel N and Rubin P 2001 *Physica* C **356** 171
[127] Entin-Wohlman O and Imry Y 1988 *Physica* C **153–155** 1323
Entin-Wohlman O and Imry Y 1989 *Phys. Rev.* B **40** 6731
[128] Vasil'chenko A N and Sokol A V 1989 *Zh. Eksp. Teor. Fiz.* **96** 377 *Sov. Phys. JETP* **69** 213
[129] Moskalenko V A, Palistrant M E, Vakalyuk V M and Padure I V 1989 *Solid State Commun.* **69** 747
Moskalenko V A, Palistrant M E and Vakalyuk V M 1991 *Sov. Phys.—Usp.* **34** 717
[130] Volovik E G 1989 *JETP Lett.* **49** 790
[131] Friedberg R and Lee T D 1989 *Phys. Rev.* B **40** 6745
[132] Hofmann U, Keller J and Kulic M L 1990 *Z. Phys.* **81** 25
[133] Abrikosov A A 1994 *Physica* C **182** 191
Abrikosov A A and Klemm R A 1992 *Physica* C **191** 224
[134] Hirsh J E and Marsiglio F 1991 *Phys. Rev.* B **43** 424
Marsiglio F 1992 *J. Low Temp. Phys.* **87** 659
[135] Langmann E 1992 *Phys. Rev.* B **46** 9104
[136] Bulut N, Scalapino D J and Scalettar R T 1992 *Phys. Rev.* B **45** 5577
[137] Bianconi A 1993 High T_c superconductors made by metal heterostructures at the atomic limit *UK Patent Specification* 0733271 (priority date: 07 12 1993)
Bianconi A 1993 Process of increasing the critical temperature T_c of a bulk superconductor by making metal heterostructures at the atomic limit *US Patent Specification* 6, 265, 019 B1 (priority date: 07 12 1993)
Bianconi A, Missori M, Saini N L, Oyanagi H, Yamaguchi H, Ha D H and Nishiara Y 1995 *J. Supercond.* **8** 545
Bianconi A 1994 *Solid State Commun.* **89** 933
Bianconi A and Missori M 1994 *J. Physique* I **4** 361
[138] Bussmann-Holder A, Genzel L, Simon A and Bishop A R 1993 *Z. Phys.* B **91** 271
Bussmann-Holder A, Genzel L, Simon A and Bishop A R 1993 *Z. Phys.* **92** 149
[139] Chakraverty B K 1993 *Phys. Rev.* B **48** 4047
[140] Mazin I I, Liechtenstein A I, Rodriguez C O, Jepsen O and Andersen O K 1993 *Physica* C **209** 125
[141] Genzel L, Bauer M, Habermeier H-U and Brandt E H 1993 *Z. Phys.* B **90** 3
[142] Golubov A A, Dolgov O V, Maksimov E G, Mazin I I and Shulga S V 1994 *Physica* C **235–240** 2383
Golubov A A and Mazin I I 1995 *Physica* C **243** 153
Golubov A A and Mazin I I 1997 *Phys. Rev.* B **55** 15146
[143] Asomba G C 1995 *Physica* C **244** 271
Asomba G C 1995 *Physica* C **245** 355
[144] Combescot R and Leyronas X 1995 *Phys. Rev. Lett.* **75** 3732
Combescot R and Leyronas X 1996 *Phys. Rev.* B **54** 4320
Leyronas X and Combescot R 1997 *Physica* C **290** 215
Combescot R 1998 *Phys. Rev.* B **57** 8632
Combescot R 1999 *Phys. Rev. Lett.* **83** 3766
[145] Wu W C and Griffin A 1995 *Phys. Rev. Lett.* **74** 158
[146] Richardson C F and Ashcroft N W 1996 *Phys. Rev.* B **54** R764
[147] Agterberg D F, Barzykin V and Gor'kov L P 1999 *Europhys. Lett.* **48** 449
[148] Kulic M L and Dolgov O V 1999 *Phys. Rev.* B **60** 13062
[149] Imada M and Kohno M 2000 *Phys. Rev. Lett.* **84** 143
[150] Granath M, Oganesyan V, Kivelson S A, Fradkin E and Emery V J 2001 *Phys. Rev. Lett.* **87** 167011
[151] Basu S, Callan-Jones A and Gooding R J 2002 *Phys. Rev.* B **66** 144507
[152] Tanaka J 2002 *Phys. Rev. Lett.* **88** 017002
[153] Imada M 2001 *J. Phys. Soc. Japan* **70** 1218
[154] Fuchs G, Drechsler S-L, Shulga S V, Handstein A, Narozhnyi V, Nenkov K and Mueller K-H 2002 *High Temperature Superconductors* vol 41, ed A Narlikar (New York: Nova Science) pp 171–98
[155] Yamaji K 2001 *J. Phys. Soc. Japan* **70** 1476
[156] Kristoffel N and Ord T 2001 arXiv:cond-mat/010353626 March
Örd T and Kristoffel N 2002 *Physica* C **370** 17
[157] Liu A Y, Mazin I I and Kortus J 2001 *Phys. Rev. Lett.* **87** 087005

[158] Chen X K, Konstantinovic M J, Irwin J C, Lawrie D D and Franck J P 2001 *Phys. Rev. Lett.* **87** 157002
[159] Annett J F and Kruchinin S 2002 *New Trends in Superconductivity* (Dodrecht: Kluwer–Academic)
[160] Haase T and Yamaji K 2001 *J. Phys. Soc. Japan* **70** 2376
[161] Bianconi A 2002 *Int. J. Mod. Phys.* B **16** 1591
[162] Bianconi A *et al* 2002 *Phys. Rev.* B **65** 174515
[163] De La Pena O, Aguayo A and de Coss R 2002 *Phys. Rev.* B **66** 012511
[164] Brinkman A, Golubov A A, Rogalla H, Dolgov V O, Kortus J, Kong Y, Jepsen O and Andersen O K 2002 *Phys. Rev.* B **65** 180517
[165] Mazin I I, Andersen O K, Jepsen O, Dolgov O V, Kortus J, Golubov A A, Kuz'menko A B and van der Marel D 2002 *Phys. Rev. Lett.* **89** 107002
[166] Choi H J, Roundy D, Sun H, Cohen M L and Louie S G 2002 *Phys. Rev.* B **66** 020513
Choi H J, Roundy D, Sun H, Cohen M L and Louie S G 2002 *Nature* **418** 758
[167] Gonnelli R S, Daghero D, Ummarino G A, Stepanov V A, Jun J, Kazakov S M and Karpinski J 2002 *Phys. Rev. Lett.* **89** 247004
[168] Nagao H, Kruchinin S P, Yaremko A M and Yamaguchi K 2002 *Int. J. Mod. Phys.* B **16** 3419
[169] Bussmann-Holder A and Bianconi A 2003 *Phys. Rev.* B **67** 132509
[170] Mazin I I and Antropov V P 2003 *Physica* C **385** 49
[171] Schechter M, von Delft J, Imry Y and Levinson Y 2003 *Phys. Rev.* B **67** 064506
[172] Rodriguez-Nunes J J and Schmidt A A 2003 *Phys. Rev.* B **68** 224512
[173] Samuely P, Holanov Z, Szabu P, KacmarcÌk J, Ribeiro R A, Bud'ko S L and Canfield P C 2003 *Phys. Rev.* B **68** 020505
[174] Schmidt H, Gray K E, Hinks D G, Zasadzinski J F, Avdeev M, Jorgensen J D and Burley C J 2003 *Phys. Rev.* B **68** 060508
[175] Putti M, Affronte M, Manfrinetti P and Palenzona A 2003 *Phys. Rev.* B **68** 094514
[176] Dahm T and Schopohl N 2003 *Phys. Rev. Lett.* **91** 017001
[177] Agrestini S *et al* 2004 *Phys. Rev.* B **70** 134514
[178] Ummarino G A, Gonnelli R S, Massidda S and Bianconi A 2004 *Physica* C **407** 121
[179] Binnig G, Baratoff A, Hoenig H E and Bednorz J G 1980 *Phys. Rev. Lett.* **45** 1352
[180] Agterberg D F, Rice T M and Sigrist M 1997 *Phys. Rev. Lett.* **78** 3374
Agterberg D F 1999 *Phys. Rev.* B **60** 749
[181] Zhitomirsky M E and Rice T M 2001 *Phys. Rev. Lett.* **87** 057001
[182] Shulga S V, Drechsler S L, Fuchs G, Muller K H, Winzer K, Heinecke M and Krug K 1998 *Phys. Rev. Lett.* **80** 1730
[183] Boaknin E *et al* 2003 *Phys. Rev. Lett.* **90** 117003
[184] Bianconi A 2006 *Phys. Status Solidi* a **203** 2950–5
[185] Bianconi A 2006 *J. Phys. Chem. Solids* **67** 566–9